

**Dept. of Mathematics**  
**Dr. Bhimrao Ambedkar University, Agra**  
**M.Sc. (Mathematics) as per NEP-2020**

**C–1: Advance Abstract Algebra**

Recall: Groups, Subgroups, Cyclic Groups, Permutation Groups, Cayley's Theorem, Lagrange's Theorem, Normal subgroups, Quotient Groups, Homomorphism, Isomorphism Theorems, Direct Products. Conjugate elements and Class Equation of Finite groups Cauchy's theorem, Sylow's Theorems, Polynomial Rings, Quotient Rings, Ideals, Maximal Ideals, Prime ideals, integral domains, Unique Factorization domains, Principal Ideal Domains Euclidean Domains. Fields and Field extensions, Galois Fields, Solvability of Polynomials by Radicals.

**C–2: Ordinary Differential Equations and Partial Differential Equations**

Ordinary Differential Equations (ODEs): Existence and uniqueness of solutions of initial value problems for first-order ordinary differential equations, singular solutions of first-order ODEs, and the system of first-order ODEs. The general theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs): Lagrange and Charpit methods for solving first-order PDEs, Cauchy problem for first-order PDEs. Classification of second-order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

**C–3: Probability and Statistics**

Probability: Baye's theorem, probability mass and density function, distribution function, moments generating function.

Probability distribution: Uniform, Binomial, Hypergeometric, Poisson, Rectangular, Gamma, Beta, Exponential, Normal.

Test of significance based on  $\chi^2$ .

Sampling (Large sample), test of significance based on t, F and Z.

**C–4: Computational Numerical Methods**

Error analysis, Numerical solution of algebraic and transcendental equations, Newton-Raphson method, fixed point iteration.

Interpolation: existence and error of polynomial interpolation, Lagrange, Newton, Hermite (Oscillatory), cubic spline Interpolations, Numerical differentiation and integration, Trapezoidal and Simpson rule, Gaussian quadrature; (Gauss-Legendre and Gauss-Chebyshev).

Numerical solution of systems of linear equations: direct and iterative methods, (Jacobi Gauss-Seidel and SOR) with convergence.

Matrix eigenvalue problems: Jacobi and Given's methods. Numerical solution of ODE: initial value problems, Taylor series method, Runge-Kutta methods, predictor-corrector methods, Numerical solutions of PDE (finite difference method), Schmidt, Lassonnen, Crank-Nicolson, Richardson, DuFort and Frankel Methods. Convergence and stability

## **C-5: Minor**

### **C-6: Real Analysis**

Dedekind's Theory: Real rational and real irrational numbers, ordering of real numbers, Archimedean property, Trichotomy law, insufficiency of rational numbers.

Riemann Stieltjes Integration: Riemann integration as a special case of Riemann Stieltjes integration, Darboux's theorem, Algebra of integrable functions, Continuity and differentiability of the integral function, Fundamental theorem of Calculus.

Uniform Convergence: Sequences and series of functions, pointwise and uniform convergence, Abel and Dirichlet tests for uniform convergence, Uniform convergence and continuity, Uniform Convergence and Riemann integration, Uniform convergence and Differentiation, Cauchy criterion for Uniform convergence, Weierstrass M – test for uniform convergence.

Euclidean Spaces: Algebraic structure of  $\mathbb{R}^n$ , Geometrical representation of  $\mathbb{R}^n$ , Triangle inequality, Neighborhood of a point, limit point of a set, Open and closed sets in  $\mathbb{R}^n$ , Bounded and compact sets in  $\mathbb{R}^n$ , Bolzano Weierstrass Theorem, Heine Borel Theorem, Sets with Heine Borel Property.

Real valued functions of several real variables: Limit and continuity of functions of several variables, Algebra of continuous functions, compactness preserving the character of continuous function, Partial derivatives, Partial Derivatives of higher order, Schwarz's and Young's theorem.

### **C-7: Functional Analysis**

Linear spaces, Dimension of a linear space.

Norm and Normed linear spaces, Completeness, Banach Spaces, Convex sets in Normed linear spaces, Quotient spaces.

Linear Operators: Elementary properties, Linear operators in finite dimensional space, Spaces of continuous linear operators, Boundedness and continuity of linear operators.

Inner product space: Simple properties, Hilbert space, Orthogonal complements, Projections, Orthonormal Basis, Bessel's inequality, Parseval identity, Self-adjoint operators, Normal operators and Unitary operators. Hahn-Banach theorem, Open mapping theorem, Closed graph theorem and Spectral theorem for normal operators.

### **C-8: Mathematical Modeling**

Role of Mathematics in problem-solving, Problem definitions, System

Characterizations, Mathematical Modeling, Mathematical formulations, Analysis of MF (Dynamic, Static, and Stochastic formulations and its analysis), Simulations, Parameter Estimations, Design of Experiments, Validations.

### **C-9: Inventory and Queueing Theory**

Inventory policy, The concept of EOQ, Deterministic inventory models with shortage and no shortage, multi-items deterministic problems, Dynamic order Quantity, Inventory problem with uncertain demand, One Periodic problem without a setup cost and with setup cost.

Queueing system, Queueing models, Birth-Death models,  $M/M/1:\infty$ ,  $M/M/1:N$ ,  $M/M/C:\infty$   $M/M/C:N$ ,  $M/E_k/1:\infty$ ,  $M/G/1:\infty$ .

### **C-10: Practical**

Problems of Numerical Methods & Operations Research through 'C' and 'C++'.  
Numerical Integration, Solution of ODE, Solution of System of Linear Equations,  
Solutions of LPP, Solution of Inventory Problems

### **C-11: Topology**

Definitions and examples of topological spaces, closed sets, closure of a set, dense subsets, Kuratowski closure axioms, neighbourhood of a point, interior, exterior, frontier (Boundary), accumulation (limit) point and derived sets, the basis for topology, order topology, subspace topology, continuous functions and homeomorphism, product topology.

Connected spaces, connected subspaces of real line, components and locally connected spaces, compact spaces, compact subspaces of the real line.

Countability axioms, separation axioms, normal spaces.

Urysohn's lemma, Urysohn's metrization theorem, Tychonoff's theorem, Stone-Cech compactification (Statement only).

### **C-12: Fuzzy Sets and Fuzzy Logics**

Fuzzy sets: Basic definitions,  $\alpha$ -level sets, convex fuzzy sets, basic operations on fuzzy set types of fuzzy sets, Cartesian products, algebraic products, bounded sum and difference, norm and t-conorms. Intuitionistic fuzzy set and hesitant fuzzy set.

The extension principle. Zadeh's extension principle of fuzzy sets, image and inverse of fuzzy sets. Fuzzy numbers, elements of fuzzy arithmetic.

Fuzzy relations and Fuzzy Graphs: Fuzzy relations of fuzzy sets, the composition of fuzzy relations, min-max composition and its properties, fuzzy equivalence relations, fuzzy compatibility relations, fuzzy relation equivalences, fuzzy graphs, similarity relation.

Possibility Theory: fuzzy measure, evidence theory, necessity measure, possibility measure, possibility distribution, possibility theory and fuzzy sets, possibility theory versus probability theory.

Fuzzy logic: An overview of classical logic, multi-valued logic, fuzzy propositions, fuzzy quantifiers, linguistic variables and hedges, inference from conditional fuzzy propositions, and the compositional rule of inference.

### **C-13: Mathematical Programming**

Fundamental properties of solutions for linear programming, Revised simplex method, Parametric linear programming, Linear fractional programming, Dual simplex method, Integer programming, Post optimality analysis, Non-linear programming, Kuhn-Tucker conditions of optimality, Quadratic programming, method due to Beale, Wolfe and duality in Quadratic programming.

### **C-14: Elective-I**

#### **Discrete Mathematics**

Formal Logic: Statements, Symbolic representation and tautologies, Quantifiers, Predicates and Validity, Propositional Logic.

Lattices: Lattices as partially ordered sets, their properties, Lattices as algebraic system, Sub-lattices, Direct products, and homomorphisms, some special lattices, e.g. complete, complemented and distributive lattices.

Boolean Algebra: Boolean Algebras, as Lattices, Various Boolean identities, The switching algebra example, subalgebras, direct products and homomorphisms, join – irreducible elements, sums and minterms, Boolean forms and their equivalence, minterm Boolean forms, Sum of products canonical forms, minimization of Boolean functions, application of Boolean algebras to switching theory, The Karnaugh map method.

Graph Theory: Definition of (undirected) Graphs, Path, Cycles, and subgraphs, induced subgraphs, degree of vertex, connectivity, planar graphs and their properties, trees, Euler's formula for connected planar graphs, complete and complete bipartite graphs Kuratowski's theorem (Statement only) and its use, spanning trees, cut-sets, fundamental cut-sets, and circuits, minimal spanning trees and Kruskal's algorithm, matrix representations of graphs, Euler theorem on the existence of Eulerian paths and circuits, directed graphs, in-degree and out-degree of a vertex, weight undirected graphs, Dijkstra's algorithm, strong connectivity and Warshall algorithm, directed trees, search trees, tree traversals.

### **Financial Mathematics**

Financial market and derivatives. Binomial Model: Binomial and CRR model, pricing a European and American contingent claim. Finite market model: Definition, First and second fundamental theorem of Asset pricing, pricing European contingent claims. Incomplete markets, Separating hyperplane theorem.

Black-Scholes model, Equivalent martingale measure. European call option-Black-Scholes formula. American call and put option.

Multi-dimensional Black-Scholes model: First fundamental theorem of asset pricing. Form of equivalent local martingale measures. Second fundamental theorem of Asset pricing. Pricing European contingent claims. Incomplete markets.

### **Reliability Theory**

Reliability Definition, Failure data analysis, Hazard Models, System Reliability. Series, Parallel, Mixed configuration complex systems, MTBF, Markov models. Reliability Improvement, Redundancy optimization, fault Tree Analysis, Tie set Cutset, Reliability using Boolean Algebra, Maintenance and Availability, Repairable System. Reliability allocation application. Application of Reliability theory.

### **Coding Theory**

Introduction to coding theory, detection and correction of errors, MLD, error-correcting coding and detecting coding. Linear codes, algebraic definitions and concepts, generating and parity check matrices, property of linear codes, MLD for linear codes, Perfect and related codes, some bounds for codes, Hamming codes, extended codes, extended Golay codes, Golay codes, Reed-Muller code, Cyclic linear codes, polynomials representation of words, generating parity check matrices for cyclic codes, Dual cyclic codes, BCH codes, Finite fields, Cyclic Hamming codes, Reed Solomon, codes over  $GF(2^r)$ .

### **Summability Theory**

Special methods of summation: Nörlund means, Regularity and consistency of Nörlund means, Inclusion, Equivalence, Euler means, Regularity of  $(E,1)$  method, Abelian means, Regularity of  $(A,\lambda)$  method, A-method and its regularity, A Theorem of inclusion for Abelian means, Complex methods, Summability of  $1-1+1-\dots$  by special Abelian Methods, A theorem of consistency, Methods ineffective for the series  $1-1+1-\dots$ , Riesz's Typical means.

Arithmetic means: Hölder's means, Simple theorems concerning Hölder Summability, Cesàro means, Simple theorems concerning Cesàro summability, Cesàro and Abel summability, Cesàro means as Nörlund means, Equivalence Theorem, Reisz's arithmetic means, Uniformly distributed sequence, Tauberian theorems for Cesàro summability.

The Methods of Euler and Borel: The  $(E,q)$  method, Simple properties of the  $(E,q)$  method, The formal relation between Euler's and Borel's methods, Borel's Methods, Normal, absolute and regular summability, Abelian Theorems for Borel's summability.

## **C-15 Practical**

### **Computer Programming with 'MATLAB'**

Problems of Operations Research through 'MATLAB'.

Problems of Numerical Methods through 'MATLAB'.

### **C-16: Complex Variables**

Analytic Functions, Power series, absolute convergence, Cauchy Hadamard Theorem.

Bilinear transformations, angle preserving property of Analytic mappings, Isogonal and Conformal mapping.

Complex integration: integral along an oriented curve, Sufficient condition for integrability, ML inequality, Cauchy Goursat Theorem for simply connected and multiply connected domains, Cauchy integral formula, Poisson's integral formula, Taylor's theorem, Morera's Theorem, Liouville's Theorem, Theorem of the Arithmetic Mean, Maximum Modulus Principle.

Singular points: Isolated character of zeros of an analytic function, Laurent series, absolute convergence and analyticity of Laurent's series, Laurent's theorem, Removable singularity, Pole and essential singularity, Behavior of a function near an isolated essential singularity, behavior at infinity.

Meromorphic Functions: Definition, characterization of Polynomials as Entire functions and Rational functions as Meromorphic functions, Residue at an isolated singularity Residue at infinity, Residue Theorem, number of zeros and poles in a domain, Principle of argument, Rouche's theorem, Fundamental theorem of Algebra.

Calculus of Residues: Evaluation of some real integrals.

### **C-17: Fluid Dynamics**

Kinematics: Lagrangian and Eulerian methods, Equation of continuity, Boundary surface, stress lines, path lines and streak lines, velocity potential and irrotational motions, vortex lines.

Equation of Motion: Lagrange's and Euler's equations of motion, Bernoulli's theorem, equation of motion of flux method, equation referred to moving axes, impulsive actions, stream function, irrotational motion in two-dimensions, complex velocity-potential,

sources, sinks, doublets and their images, conformal mapping Milne-Thompson circle theorem.

Two-dimensional irrotational motion produced by the motion of circular, co-axial and elliptic cylinders in an infinite mass of liquid, the kinetic energy of the liquid, theorem of Blasius motion of a sphere through a liquid at rest at infinity, liquid streaming past a fixed sphere, equation of motion of a sphere, Stoke's stream function.

Stress component in a real fluid, relations between rectangular components of stress, the connection between stresses and gradients of velocity, Navier-Stoke's equation of motion, plane Poiseuille and through tubes of the uniform cross-section in form of a circle, annulus unsteady flow over a flat plate.

### **C-18: Elective-II**

### **C-19: Elective-III**

#### **Number Theory and Cryptology**

Euclidean algorithm. Quadratic residues and Reciprocity. Some simple cryptosystems. DES and AES systems. Public key cryptosystems, Diffie-Hellman key exchange, RSA and ElGamal systems. Signature Schemes, Elliptic curve, Crypto systems.

#### **Calculus of Variations and Integral Equation**

Calculus of Variations: Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Linear Integral Equations: Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigen functions, resolvent kernel.

#### **Wavelet Analysis**

The Scalable Structure of Information: The New Mathematical Engineering, Good Approximations, Wavelets: A Positional Notation for Functions, Review of linear algebra: Vector spaces, basis, dimension, linear transformations, matrices and digitalization, inner products and orthonormal bases. Wavelet Theory: Algebra and Geometry of Wavelet: Matrices, One-Dimensional Wavelet Systems, Examples of One-Dimensional Wavelet Systems, Higher Dimensional, Wavelet Systems. Wavelets on  $Z$ ,  $Z_n$ ,  $l^2(Z)$ , Fourier series, transform and convolution on  $l^2$ .

Wavelet Approximation and Algorithms: The Mallat Algorithm

#### **Control Theory**

Mathematical models of control systems, State space representation, Autonomous and non-autonomous systems, State, transition matrix, Peano series Solution of a linear dynamical system. Block diagram, Transfer function, Realization, Controllability, Kalman theorem, Controllability Grammian, Control computation using Grammian matrix, Observability, Duality theorems. Discrete control systems, Controllability and Observability results for discrete systems. Companion form, Feedback control, State observer, Realization. Liapunov stability, Stability analysis for linear systems, Liapunov theorems for stability and instability for nonlinear systems, Stability analysis through Linearization, Routh criterion, Nyquist criterion, Stabilizability and detachability, State feedback of multivariable system, Riccati equation, Calculus of variation, Euler-

Hamiltonian equations, Optimal control for nonlinear control systems, Computation of optimal control for linear systems. Control systems on Hilbert spaces, Semi group theory, Mild solution, Control of a linear system

## **Bio-mathematics**

Epidemic models: Deterministic models without removal, a general deterministic model with removal, a general deterministic model with removal and immigration, and control of an epidemic. Mathematical models in Pharmacokinetics: basic equations and their solutions, solutions for special cases, determination of transfer coefficients and compartment volumes, mathematical techniques used in compartment analysis, stochastic compartment models. Models for blood flow some basic concepts for fluid dynamics, basic concepts about blood, cardiovascular system and blood flows, steady non-Newtonian fluid flow in circular tubes, Newtonian pulsatile flows in rigid and elastic tubes, blood flow through the artery with mild stenosis, peristaltic flow in tubes and channels, models for air flow in lungs, Diffusion and Diffusion-reaction models, the diffusion equations, oxygen diffusion living tissues. Non-linear populations growth models, models in genetics, the basic model for inheritance, models for genetic improvement, selection and mutation, applications in ecology

## **Soft Computing**

**Neural Networks-1**(Introduction & Architecture) Neuron, Nerve structure and synapse, Artificial Neuron and its model, activation functions, Neural network architecture: single layer and multilayer feed forward networks, recurrent networks. Various learning techniques; perception and convergence rule, Auto-associative and hetro-associative memory.

**Neural Networks-II** (Backpropagation networks) Architecture: perceptron model, solution, single layer artificial neural network, multilayer perceptron model; back propagation learning methods, the effect of learning rule co-efficient; backpropagation algorithm, factors affecting backpropagation training, applications.

**Fuzzy Logic** –Basic Concepts of fuzzy sets and logic, Membership functions, interference in fuzzy logic, fuzzy if-then rules, Fuzzy implications and Fuzzy algorithms, Fuzzyfications & Defuzzificataions, Fuzzy Controller, Industrial applications.

**Genetic Algorithm(GA)** Basic concepts, working principle, procedures of GA, flow chart of GA, Genetic representations, (encoding) Initialization and selection, Genetic operators, Mutation, Generational Cycle, and applications.

## **Special Function**

Series solution of second order linear differential equations: Ordinary and singular points of a linear differential equation, The point at infinity, Series solution near regular singular point.

The Gamma and Beta functions: Definitions and basic properties.

Hypergeometric Functions: The hypergeometric series, An integral formula for the hypergeometric series, The hypergeometric equation, Linear relations between the

solutions of the hypergeometric equation, Relations of contiguity, and The confluent hypergeometric function.

Legendre Functions: Legendre polynomials, Recurrence relations for the Legendre polynomials, The formulae of Murphy and Rodrigues, Series of Legendre polynomials, Legendre's differential equation.

Bessel Functions: The origin of Bessel functions, Recurrence relations for the Bessel coefficients, Series expansion for the Bessel coefficients, Integral expressions for the Bessel coefficients, The additional formula for the Bessel coefficients, Bessel's differential equation.

Hermite Functions: The Hermite polynomials, Hermite differential equation, Hermite functions.

## **Measure Theory**

Introduction and Motivation of Measure theory, Motivation and definition of Lebesgue outer measure on  $R^n$ . Properties of Lebesgue outer measure on  $R^n$ , Caratheodory extension theorem.

Lebesgue measurability, Vitali and Cantor sets, Boolean and sigma algebras

Abstract measure spaces with examples: Borel and Radon measures, Metric outer measures, Lebesgue-Stieljes measures, Hausdorff measures and dimension.

Measurable functions and abstract Lebesgue integration, Monotone convergence theorem, Fatou's lemma, Tonelli's theorem

Borel-Cantelli Lemma, Dominated convergence theorem, the space  $L^1$ . Various modes of convergence and their interdependence.

Riesz representation theorem, examples of measures constructed via RRT. Product measures and Fubini-Tonelli theorem. Hardy-Littlewood Maximal inequality and Lebesgue's differentiation theorem.